EE 434
Lecture 19

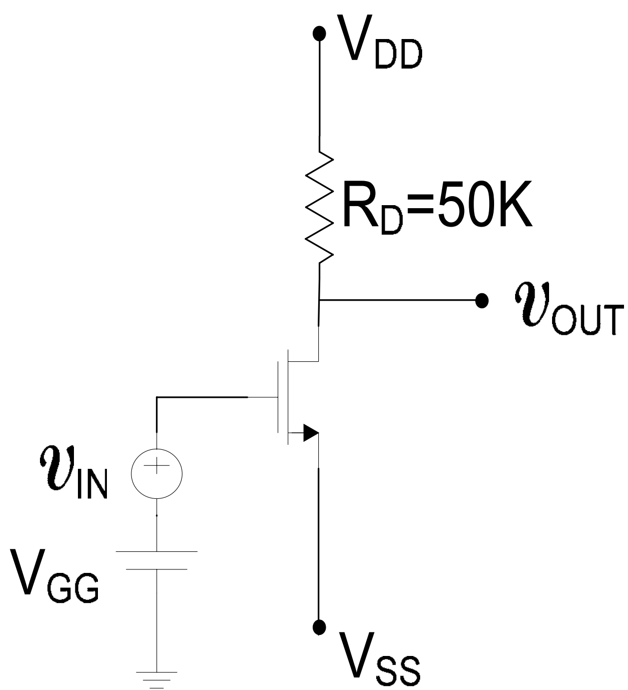
Model Extension

Small signal model extension

Small signal analysis

Quiz 13

Assume the MOS transistor in this circuit was designed in a 0.6u process with device parameters μC_{OX}=100μA/V^2 and V_T=1V. With the square-law model of the device introduced in class, it can be shown that the small-signal gain for this circuit is $\mathbf{A_V = -g_m R_D}$ where g_m is the transconductance gain of the transistor. Assume a reverse-engineering team is trying to determine what the dimensions are of the device and can not open the package to see the device. They did, however, measure the small signal voltage gain and it was -4 and they measured the quiescent VGS and it was 3V. What is W/L for the transistor?



And the number is ….

1 8 7 5 3

6 9 4 2

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6

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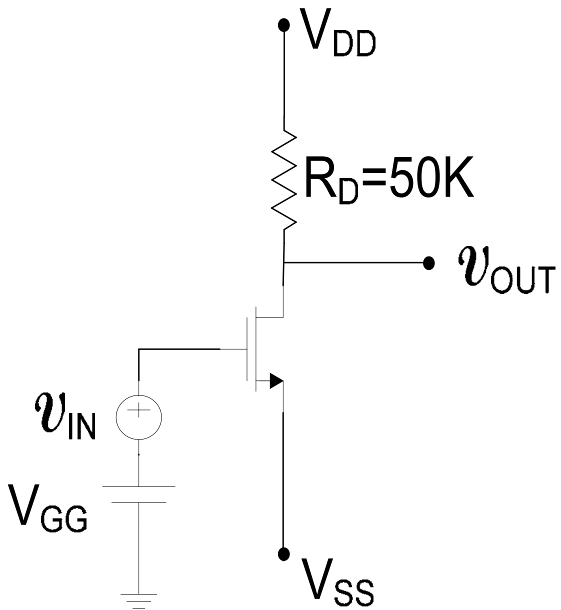

Solution:

$$\mathbf{g_m = -\frac{A_V}{R_D} = \frac{--4}{50K} = 8E-5A/V}$$

$$\mathbf{g_m = \mu C_{OX} V_{EB} \frac{W}{L}}$$

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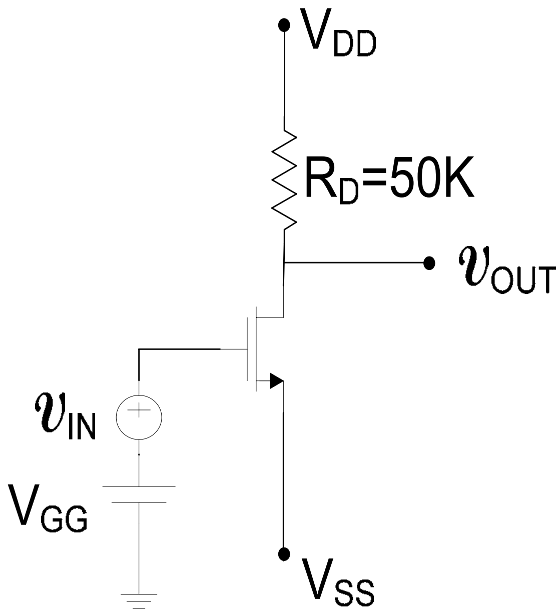

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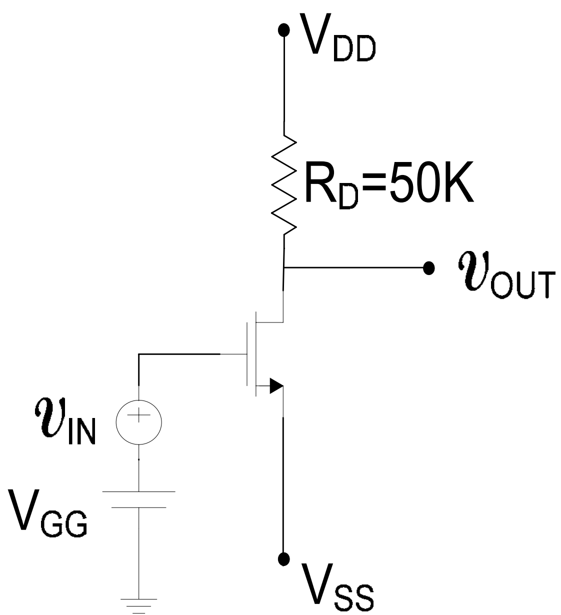

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$$\mathbf{g_m = \mu C_{OX} V_{EB} \frac{W}{L}}$$

$$\mathbf{\frac{W}{L} = \frac{g_m}{\mu C_{OX} V_{EB}}}$$

$$\mathbf{\frac{W}{L} = \frac{8E\text{-}5}{100E\text{-}6 \;\; 2V} = 0.4}$$

Review from Last Time

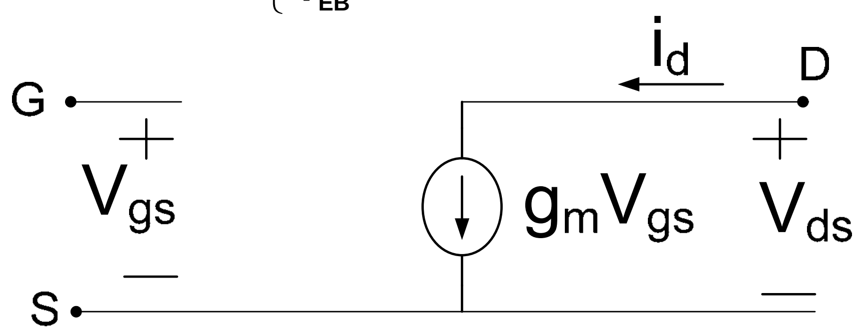
Small signal model for MOS transistor was developed

$$i_g = 0$$

$$i_b = 0$$

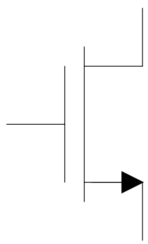
$$i_d = g_m v_{gs}$$

$$g_m = \begin{cases} \mu C_{OX}\frac{W}{L}V_{EB} \\ \sqrt{2\mu\mu_{OX}\frac{W}{L}}\sqrt{I_{DQ}} \\ \frac{2I_{DQ}}{V_{EB}} \end{cases}$$



Review from Last Time

Model Extension to Account for Slope of I_D-V_{DS} Characteristics



$$I_D = \begin{cases} 0 & V_{GS} \le V_T \\ \mu C_{OX} \frac{W}{L}\left(V_{GS} - V_T - \frac{V_{DS}}{2}\right)V_{DS} & V_{GS} \ge V_T \quad V_{DS} < V_{GS} - V_T \\ \mu C_{OX} \frac{W}{2L}\left(V_{GS} - V_T\right)^2 \bullet \left(1 + \lambda V_{DS}\right) & V_{GS} \ge V_T \quad V_{DS} \ge V_{GS} - V_T \end{cases}$$

Introduces a discontinuity between triode and saturation regions

- Does not exist in real devices
- Multiply by $1+\lambda V_{DS}$ in triode region as well in simulators

Issue of how gm varies with I_{DQ} discussed and apparent delima identified

Review from Last Time

How does g_m vary with I_{DQ}?

$$g_m = \sqrt{\frac{2\mu C_{OX} W}{L}}\sqrt{I_{DQ}}$$

Varies with the square root of I_{DQ}

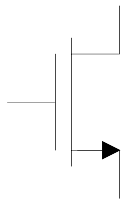
$$g_m = \frac{2I_{DQ}}{V_{GSQ} - V_T} = \frac{2I_{DQ}}{V_{EBQ}}$$

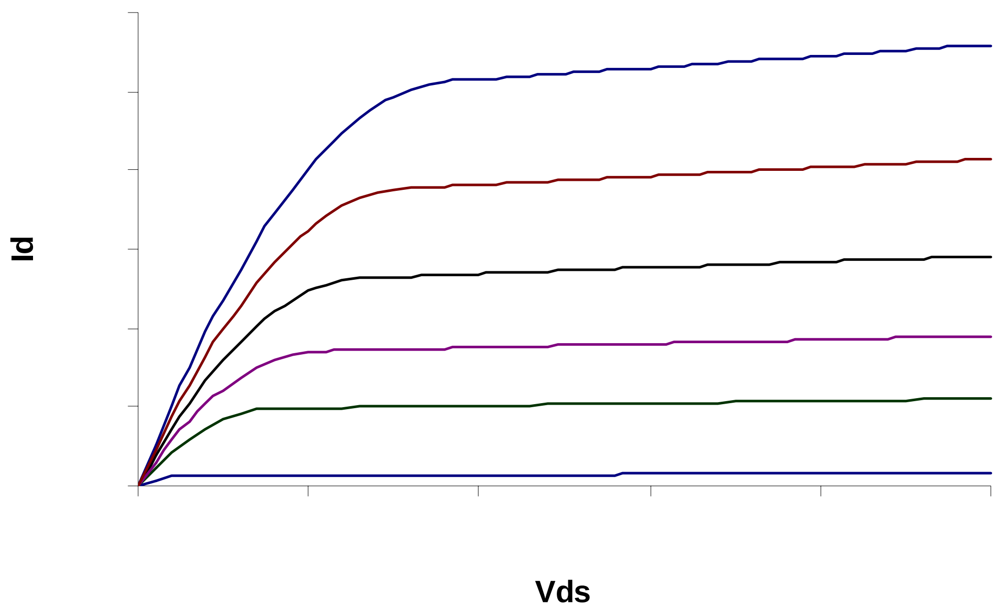
Varies linearly with I_{DQ}

$$g_m = \frac{\mu C_{OX} W}{L}\left(V_{GSQ} - V_T\right)$$

Doesn't vary with I_{DQ}

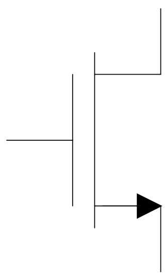
Graphical Interpretation of MOS Model

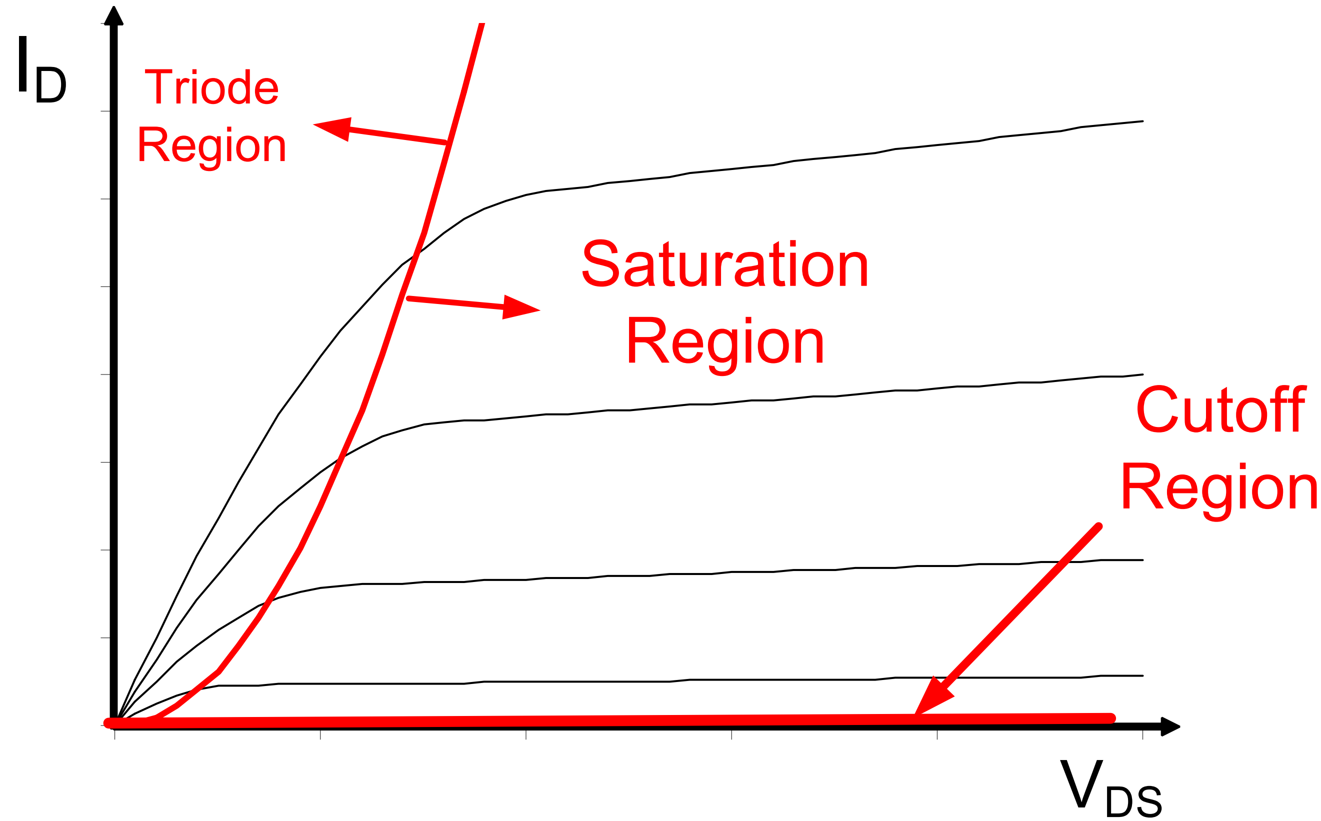




$$I_D = \begin{cases} 0 & V_{GS} \le V_T \\ \mu C_{OX} \dfrac{W}{L}\left(V_{GS} - V_T - \dfrac{V_{DS}}{2}\right)V_{DS} & V_{GS} \ge V_T \quad V_{DS} < V_{GS} - V_T \\ \mu C_{OX} \dfrac{W}{2L}(V_{GS} - V_T)^2 \bullet (1 + \lambda V_{DS}) & V_{GS} \ge V_T \quad V_{DS} \ge V_{GS} - V_T \end{cases}$$

Graphical Interpretation of MOS Model

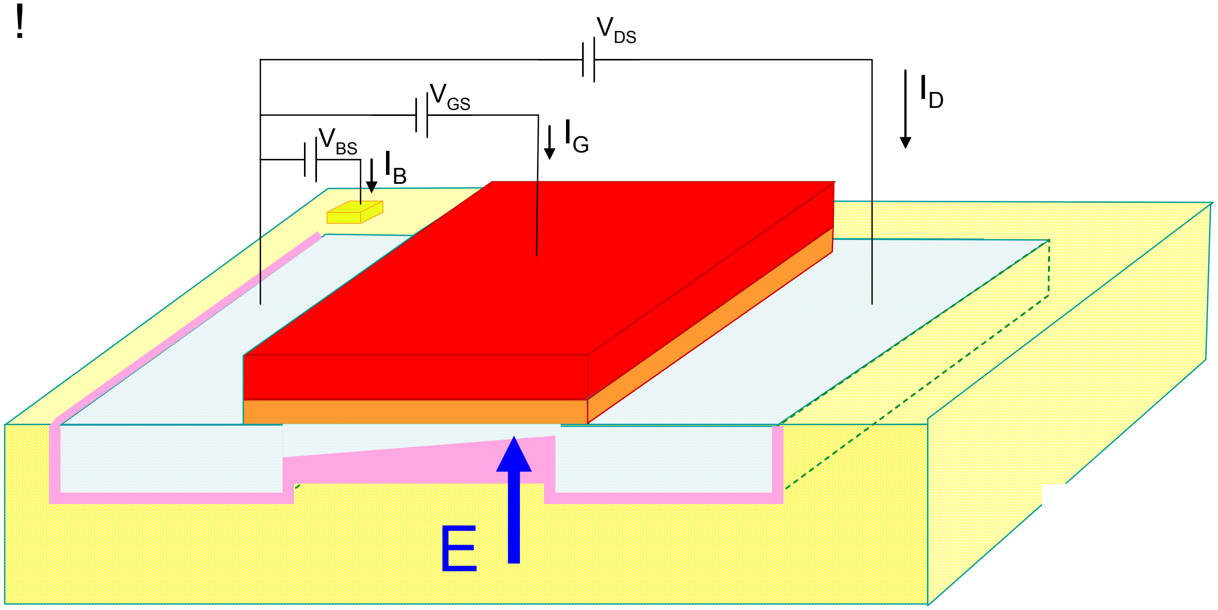




Further Model Extensions

Existing model does not depend upon the bulk voltage !

Observe that changing the bulk voltage will change the electric field in the channel region !



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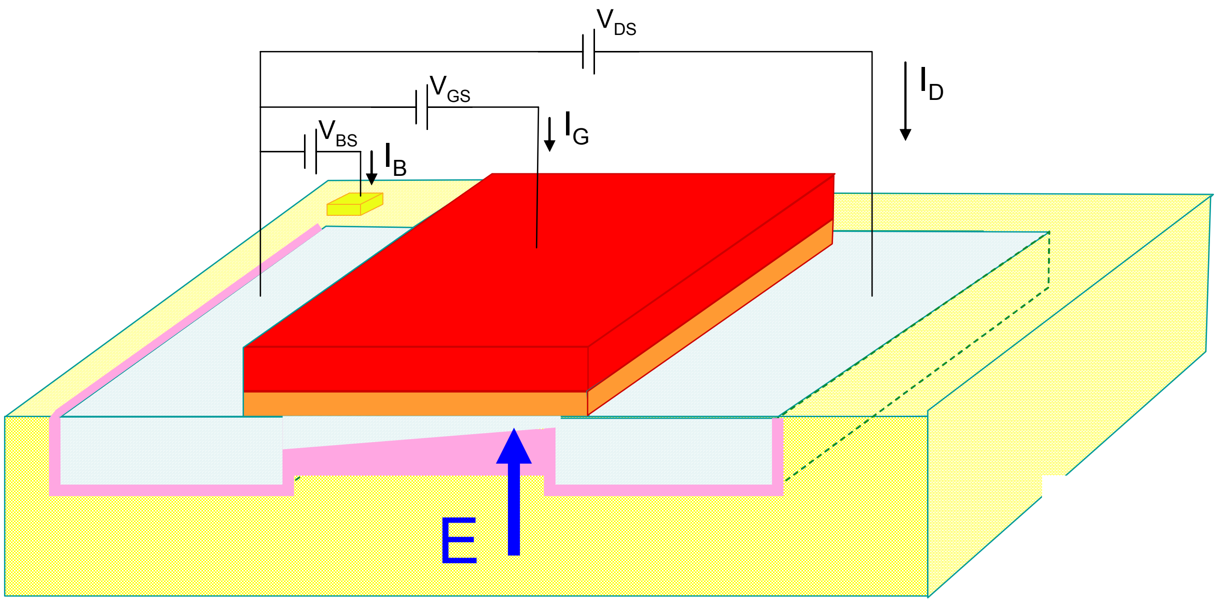


Changing the bulk voltage will change the thickness of the inversion layer

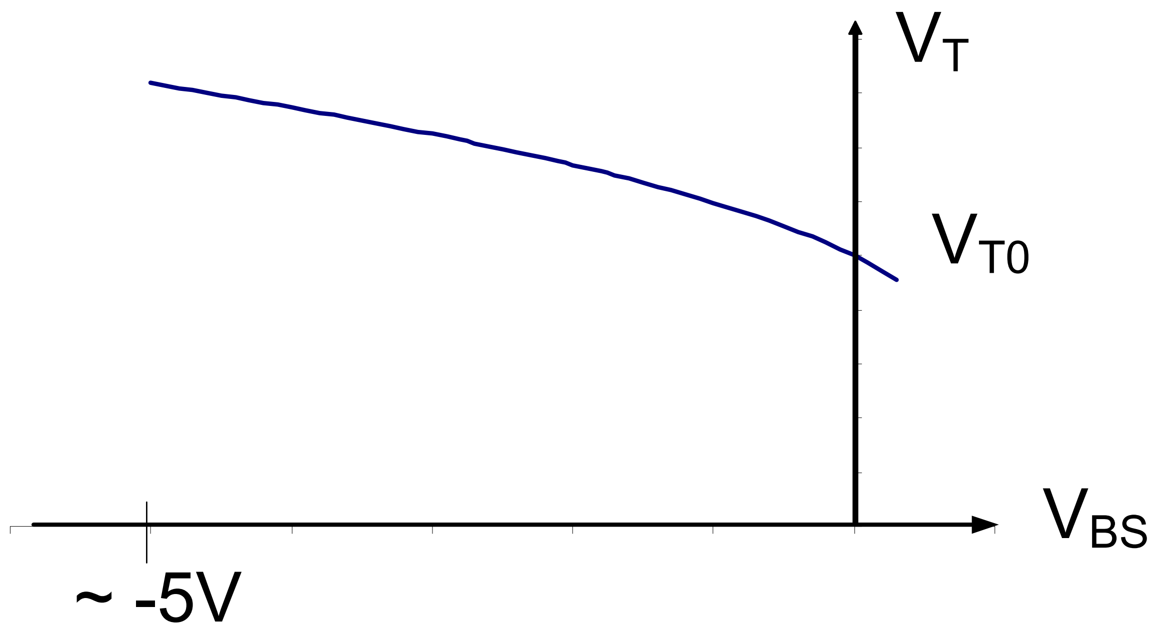
Changing the bulk voltage will change the threshold voltage of the device

$$V_T = V_{T0} + \gamma\left(\sqrt{\phi - V_{BS}} - \sqrt{\phi}\right)$$

Typical Effects of Bulk on Threshold Voltage for n-channel Device

$$V_T = V_{T0} + \gamma\left(\sqrt{\phi - V_{BS}} - \sqrt{\phi}\right)$$

$$\gamma \simeq 0.4V^{-\frac{1}{2}} \qquad \phi \simeq 0.6V$$



Bulk-Diffusion Generally Reverse Biased (V_{BS}< 0 or at least less than 0.3V) for n-channel

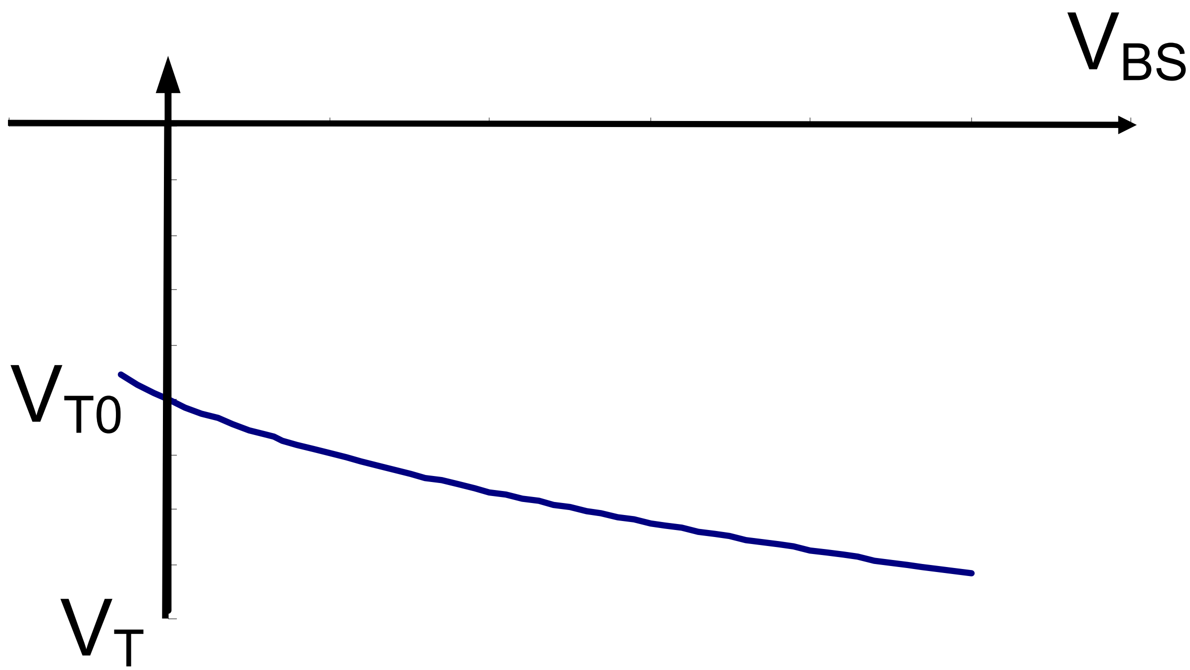
Shift in threshold voltage with bulk voltage can be substantial

Often V_{BS}=0

Typical Effects of Bulk on Threshold Voltage for p-channel Device

$$V_T = V_{T0} - \gamma\left(\sqrt{\phi + V_{BS}} - \sqrt{\phi}\right)$$

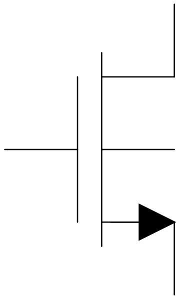
$$\gamma \simeq 0.4V^{-\frac{1}{2}} \qquad \phi \simeq 0.6V$$



Bulk-Diffusion Generally Reverse Biased (V_{BS} > 0 or at least greater than -0.3V) for n-channel

Same functional form as for n-channel devices but V_{T0} is now negative and the magnitude of V_T still increases with the magnitude of the reverse bias

Model Extension Summary



$$\mathbf{I_G = 0}$$

$$\mathbf{I_B = 0}$$

$$I_D = \begin{cases} 0 & V_{GS} \le V_T \\ \mu C_{OX}\dfrac{W}{L}\left(V_{GS} - V_T - \dfrac{V_{DS}}{2}\right)V_{DS} & V_{GS} \ge V_T \quad V_{DS} < V_{GS} - V_T \\ \mu C_{OX}\dfrac{W}{2L}\left(V_{GS} - V_T\right)^2 \bullet \left(1 + \lambda V_{DS}\right) & V_{GS} \ge V_T \quad V_{DS} \ge V_{GS} - V_T \end{cases}$$

$$V_T = V_{T0} + \gamma\left(\sqrt{\phi - V_{BS}} - \sqrt{\phi}\right)$$

Model Parameters : $\{\mu, C_{OX}, V_{T0}, \varphi, \gamma, \lambda\}$

Design Parameters : $\{W,L\}$ but only one degree of freedom W/L

Small-Signal Model Extension

$$I_G = 0$$

$$I_B = 0$$

$$I_D = \begin{cases} 0 & V_{GS} \le V_T \\ \mu C_{OX} \dfrac{W}{L}\left(V_{GS} - V_T - \dfrac{V_{DS}}{2}\right)V_{DS} & V_{GS} \ge V_T \quad V_{DS} < V_{GS} - V_T \\ \mu C_{OX} \dfrac{W}{2L}(V_{GS} - V_T)^2 \bullet (1 + \lambda V_{DS}) & V_{GS} \ge V_T \quad V_{DS} \ge V_{GS} - V_T \end{cases}$$

$$V_T = V_{T0} + \gamma\left(\sqrt{\phi - V_{BS}} - \sqrt{\phi}\right)$$

$$\mathbf{y}_{11} = \left.\frac{\partial \mathbf{I_G}}{\partial \mathbf{V_{GS}}}\right|_{\vec{V}=\vec{V}_Q} = 0 \qquad \mathbf{y}_{12} = \left.\frac{\partial \mathbf{I_G}}{\partial \mathbf{V_{DS}}}\right|_{\vec{V}=\vec{V}_Q} = 0 \qquad \mathbf{y}_{13} = \left.\frac{\partial \mathbf{I_G}}{\partial \mathbf{V_{GS}}}\right|_{\vec{V}=\vec{V}_Q} = 0$$

$$\mathbf{y}_{31} = \left.\frac{\partial \mathbf{I_B}}{\partial \mathbf{V_{GS}}}\right|_{\vec{V}=\vec{V}_Q} = 0 \qquad \mathbf{y}_{32} = \left.\frac{\partial \mathbf{I_B}}{\partial \mathbf{V_{DS}}}\right|_{\vec{V}=\vec{V}_Q} = 0 \qquad \mathbf{y}_{33} = \left.\frac{\partial \mathbf{I_B}}{\partial \mathbf{V_{GS}}}\right|_{\vec{V}=\vec{V}_Q} = 0$$

$$\mathbf{y}_{21} = \left.\frac{\partial \mathbf{I_G}}{\partial \mathbf{V_{GS}}}\right|_{\vec{V}=\vec{V}_Q} = g_m \qquad \mathbf{y}_{12} = \left.\frac{\partial \mathbf{I_G}}{\partial \mathbf{V_{DS}}}\right|_{\vec{V}=\vec{V}_Q} = g_o \qquad \mathbf{y}_{13} = \left.\frac{\partial \mathbf{I_G}}{\partial \mathbf{V_{GS}}}\right|_{\vec{V}=\vec{V}_Q} = g_{mb}$$

$$I_D = \mu C_{OX} \frac{W}{2L}(V_{GS} - V_T)^2 \bullet (1 + \lambda V_{DS})$$

$$V_T = V_{T0} + \gamma\left(\sqrt{\phi - V_{BS}} - \sqrt{\phi}\right)$$

$$g_m = \left.\frac{\partial I_D}{\partial V_{GS}}\right|_{\vec{V}=\vec{V}_Q} = \mu C_{OX} \frac{W}{\cancel{2}L}\cancel{2}\left(V_{GS} - V_T\right)^1 \bullet \left.(1 + \lambda V_{DS})\right|_{\vec{V}=\vec{V}_Q} \cong \mu C_{OX} \frac{W}{L} V_{EBQ}$$

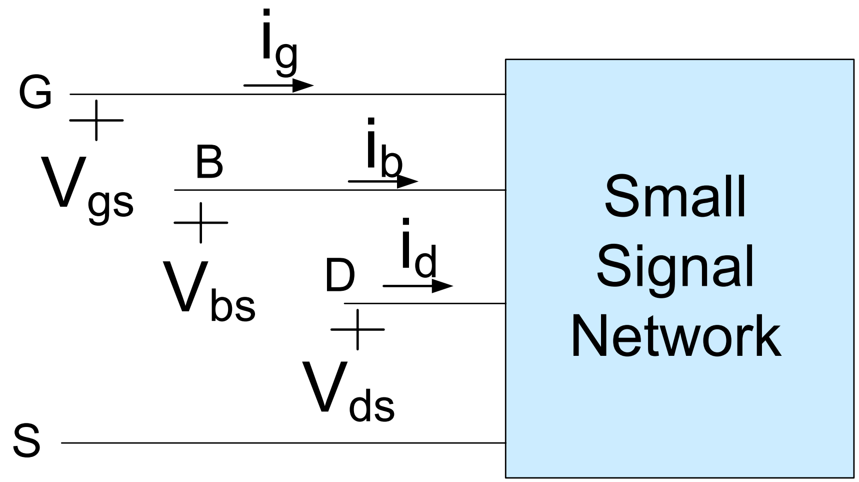
$$g_o = \left.\frac{\partial I_D}{\partial V_{DS}}\right|_{\vec{V}=\vec{V}_Q} = \mu C_{OX} \frac{W}{2L} 2\left.\left(V_{GS} - V_T\right)^2 \bullet \lambda\right|_{\vec{V}=\vec{V}_Q} \cong \lambda I_{DQ}$$

$$g_{mb} = \left.\frac{\partial I_D}{\partial V_{BS}}\right|_{\vec{V}=\vec{V}_Q} = \mu C_{OX} \frac{W}{\cancel{2}L}\cancel{2}\left(V_{GS} - V_T\right)^1 \bullet \left(-\frac{\partial V_T}{\partial V_{BS}}\right) \bullet \left.(1+\lambda V_{DS})\right|_{\vec{V}=\vec{V}_Q}$$

$$g_{mb} = \left.\frac{\partial I_D}{\partial V_{BS}}\right|_{\vec{V}=\vec{V}_Q} \cong \mu C_{OX} \frac{W}{L} V_{EB} \bullet \left.\frac{\partial V_T}{\partial V_{BS}}\right|_{\vec{V}=\vec{V}_Q} = \left(\mu C_{OX} \frac{W}{L} V_{EB}\right)(-\cancel{1})\gamma \frac{1}{2}\left.\left(\phi - V_{BS}\right)^{-\frac{1}{2}}\right|_{\vec{V}=\vec{V}_Q}(-\cancel{1})$$

$$g_{mb} \cong g_m \frac{\gamma}{2\sqrt{\phi\text{-}V_{BSQ}}}$$

Small Signal Model Summary



$$i_g = 0$$

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$$i_d = g_m v_{gs} + g_{mb} v_{bs} + g_o v_{ds}$$

$$g_m = \frac{\mu C_{ox} W}{L} V_{EBQ}$$

$$g_o = \lambda I_{DQ}$$

$$g_{mb} = g_m \left(\frac{\gamma}{2\sqrt{\phi - V_{BSQ}}} \right)$$

Small Signal Model Observation

Consider:

$$i_d = g_m v_{gs} + g_{mb} v_{bs} + g_o v_{ds}$$

3 alternate equivalent expressions for g_m

$$g_m = \frac{\mu C_{OX} W}{L} V_{EBQ} \qquad g_m = \sqrt{\frac{2\mu C_{OX} W}{L}}\sqrt{I_{DQ}} \qquad g_m = \frac{2I_{DQ}}{V_{EBQ}}$$

If $\mu C_{OX}=100\mu A/V^2$, $\lambda=.01V^{-1}$, $\gamma = 0.4V^{0.5}$, $V_{EBQ}=1V$, $W/L=1$, $V_{BSQ}=0V$

$$I_{DQ} \cong \frac{\mu C_{OX} W}{2L} V_{EBQ}^2 = \frac{10^{-4} \cancel{W}}{2\cancel{L}}(1V)^2 = 5E\text{-}5$$

$$g_m = \frac{\mu C_{OX} W}{L} V_{EBQ} = 1E\text{-}4$$

$$g_o = \lambda I_{DQ} = 5E\text{-}7$$

$$g_{mb} = g_m \left(\frac{\gamma}{2\sqrt{\phi - V_{BSQ}}} \right) = .26 g_m$$

In this example

$$g_0 << g_m, g_{mb}$$

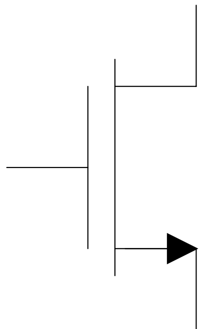
$$g_{mb} < g_m$$

This relationship is common

In many circuits, $v_{BS}=0$ as well

Small Signal Model Summary

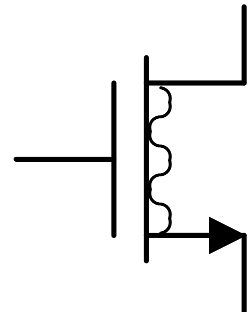
Large Signal Model



$$I_D = \begin{cases} 0 & V_{GS} \le V_T \\ \mu C_{OX} \dfrac{W}{L}\left(V_{GS} - V_T - \dfrac{V_{DS}}{2}\right)V_{DS} & V_{GS} \ge V_T \quad V_{DS} < V_{GS} - V_T \\ \mu C_{OX} \dfrac{W}{2L}\left(V_{GS} - V_T\right)^2 \bullet \left(1 + \lambda V_{DS}\right) & V_{GS} \ge V_T \quad V_{DS} \ge V_{GS} - V_T \end{cases}$$

$$V_T = V_{T0} + \gamma\left(\sqrt{\phi - V_{BS}} - \sqrt{\phi}\right)$$

Small Signal Model



$$i_g = 0$$

$$i_b = 0$$

$$i_d = g_m v_{gs} + g_{mb} v_{bs} + g_o v_{ds}$$

where

$$g_m = \frac{\mu C_{OX} W}{L} V_{EBQ}$$

$$g_{mb} = g_m\left(\frac{\gamma}{2\sqrt{\phi - V_{BSQ}}}\right)$$

$$g_o = \lambda I_{DQ}$$

How does g_m vary with I_{DQ}?

$$g_m = \sqrt{\frac{2\mu C_{OX} W}{L}}\sqrt{I_{DQ}}$$

Varies with the square root of I_{DQ}

$$g_m = \frac{2I_{DQ}}{V_{GSQ} - V_T} = \frac{2I_{DQ}}{V_{EBQ}}$$

Varies linearly with I_{DQ}

$$g_m = \frac{\mu C_{OX} W}{L}\left(V_{GSQ} - V_T\right)$$

Doesn't vary with I_{DQ}

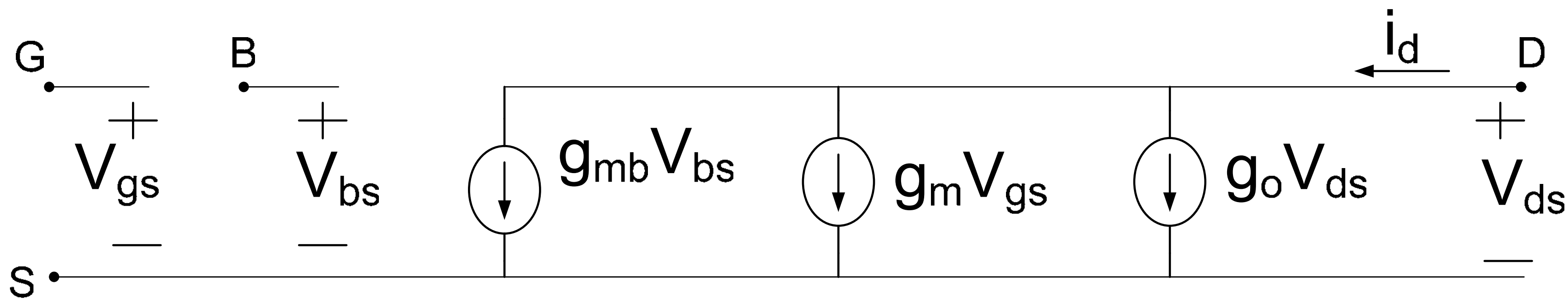
How does g_m vary with I_{DQ}?

All of the above are true – but with qualification

g_m is a function of more than one variable (I_{DQ}) and how it varies depends upon how the remaining variables are constrained

Small Signal Model Summary

An equivalent circuit



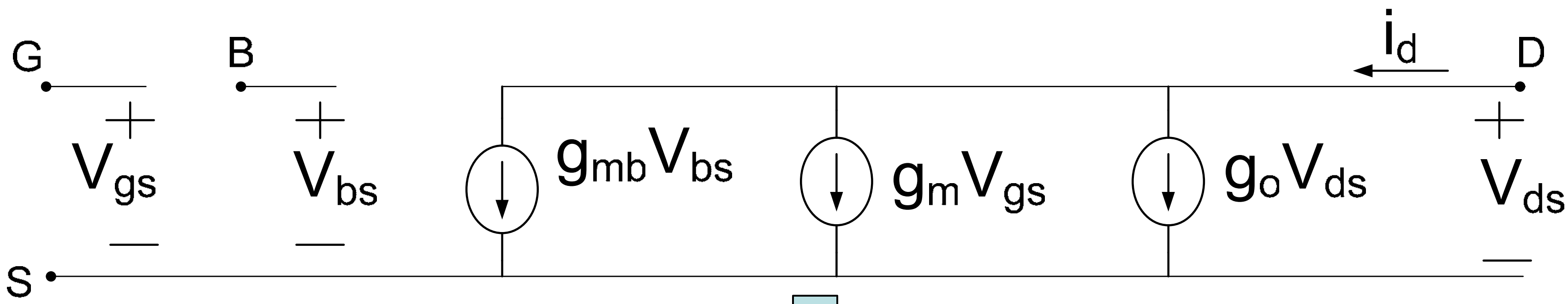
$$g_m = \frac{\mu C_{OX} W}{L}\left(V_{GSQ} - V_T\right)$$

$$g_o = \lambda I_{DQ}$$

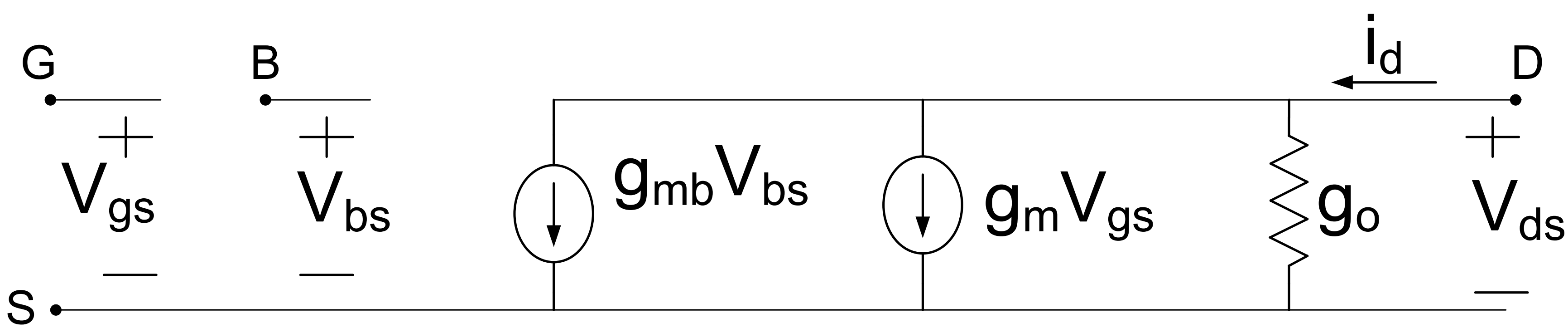
$$g_{mb} = g_m\left(\frac{\gamma}{2\sqrt{\phi - V_{BSQ}}}\right)$$

This contains absolutely no more information than the previous model

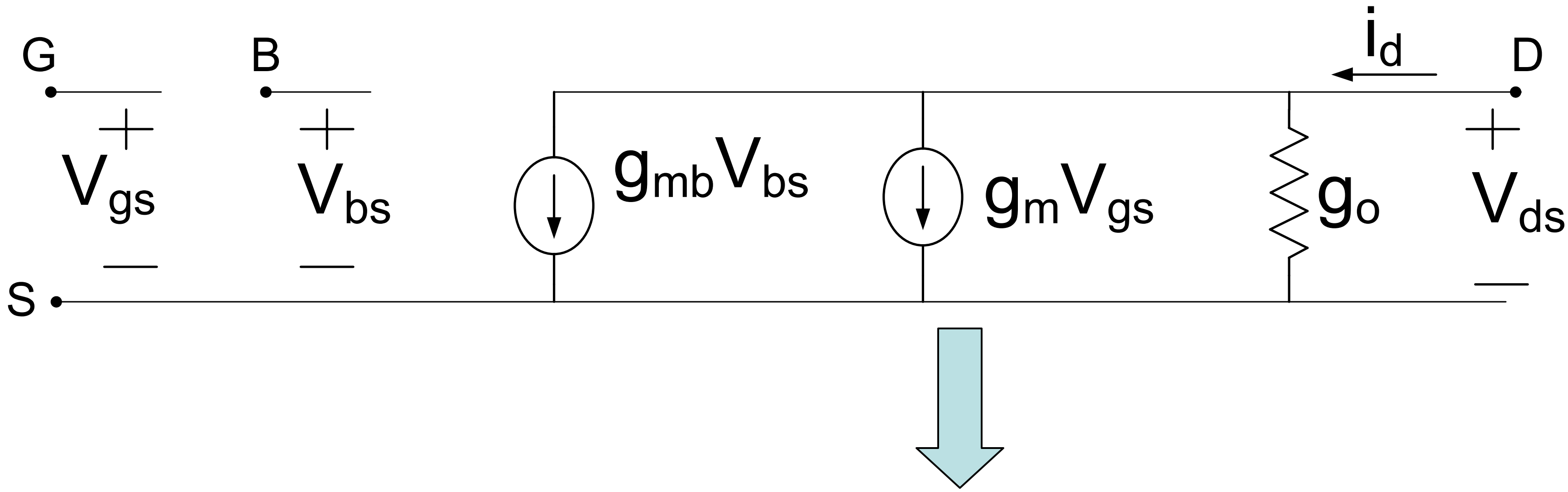
Small Signal Model Summary



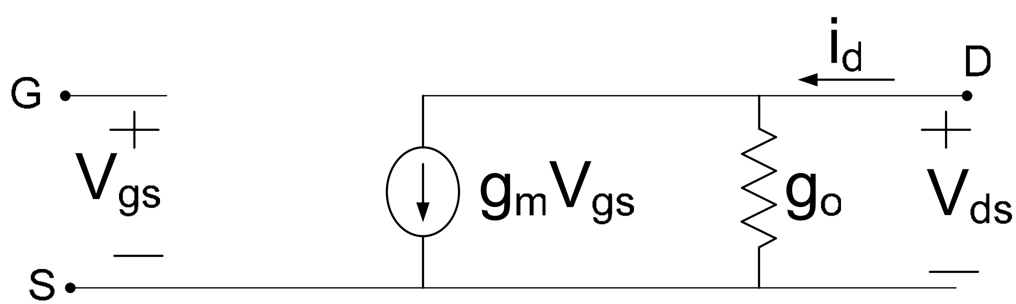
More convenient representation



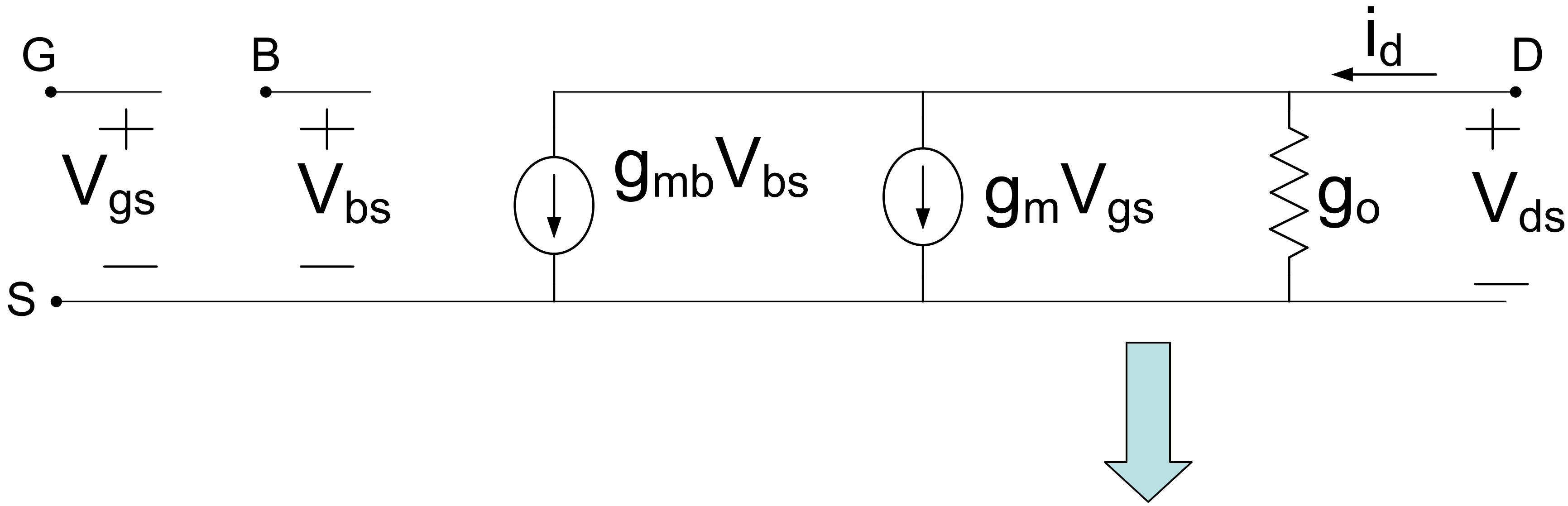
Small Signal Model Summary



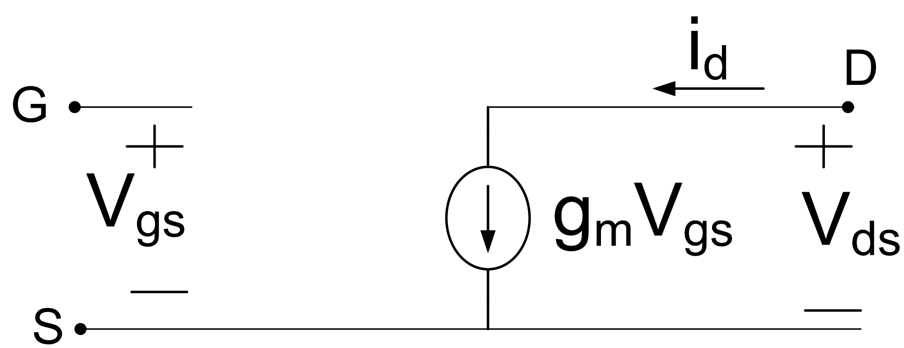
Simplification that is often adequate



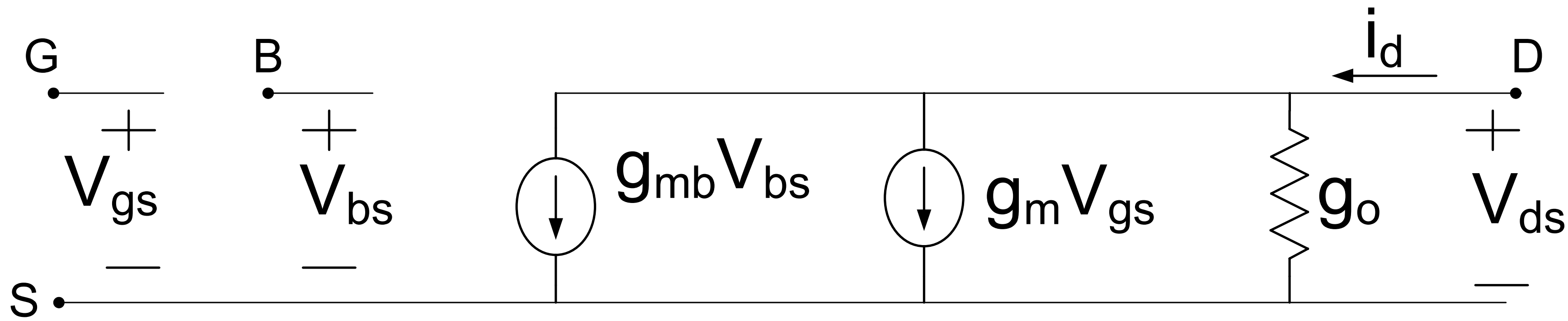
Small Signal Model Summary



Even further simplification that is often adequate



Small Signal Model Summary



Alternate equivalent representations for g_m

$$g_m = \frac{\mu C_{OX} W}{L}\left(V_{GSQ} - V_T\right) \qquad \text{from} \qquad I_D \cong \mu C_{OX}\frac{W}{2L}\left(V_{GS} - V_T\right)^2$$

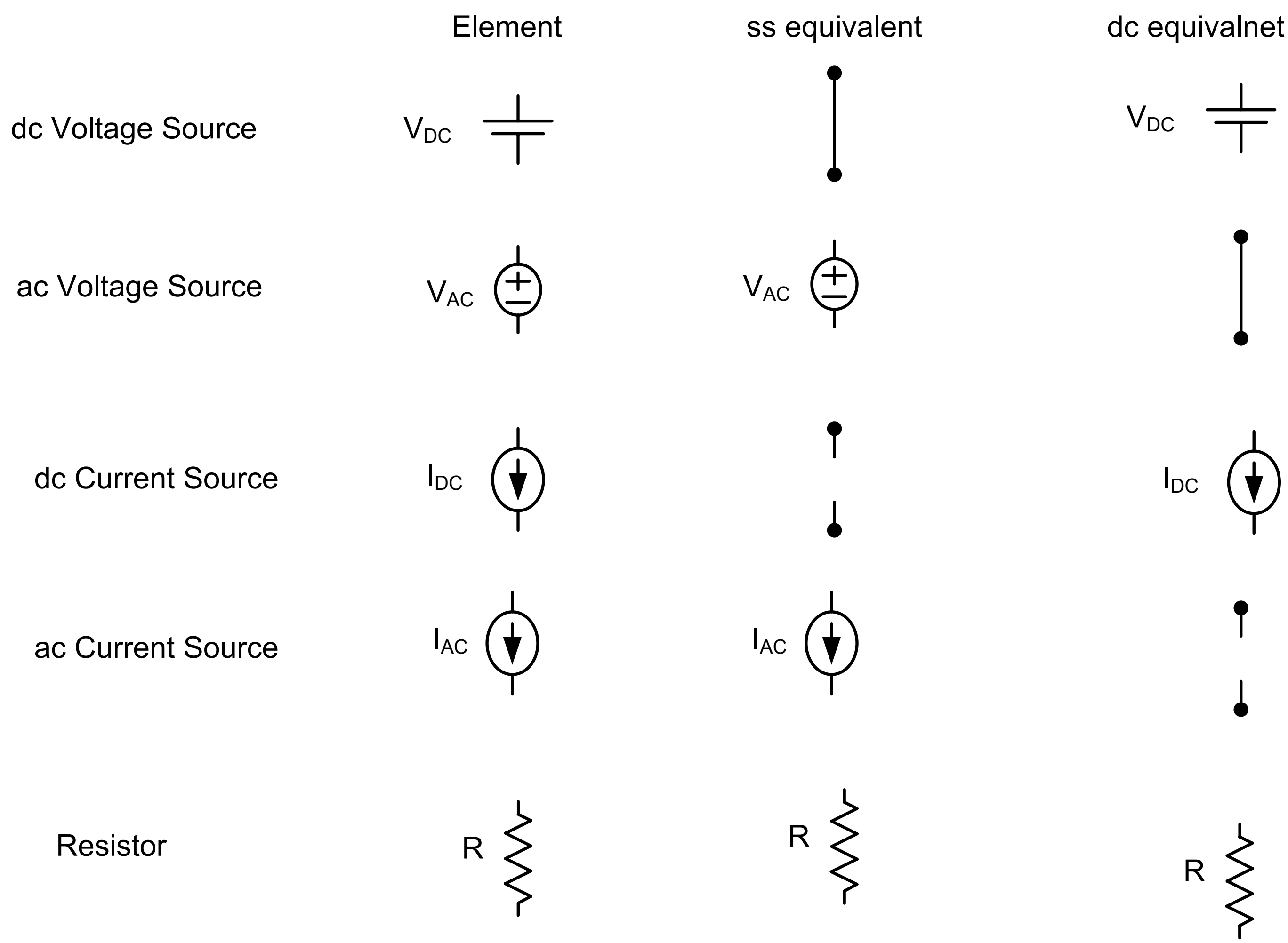
$$g_m = \sqrt{\frac{2\mu C_{OX} W}{L}}\sqrt{I_{DQ}}$$

$$g_m = \frac{2I_{DQ}}{V_{GSQ} - V_T} = \frac{2I_{DQ}}{V_{EBQ}}$$

Small-Signal Circuit Analysis

- Obtain dc equivalent circuit by replacing all elements with large-signal (dc) equivalent circuits
- Obtain dc operating points (Q-point)
- Obtain ac equivalent circuit by replacing all elements with small-signal equivalent circuits
- Analyze linear small-signal equivalent circuit

Dc and small-signal equivalent elements



Dc and small-signal equivalent elements